# Multi-criteria Mathematical Model for Partial Double Track Railway Scheduling in Urban Rail Network

Erlangga Bayu Setyawan, Dida Diah Damayanti, Anton Abdulbasah Kamil
Department of Industrial Engineering, Telkom University, Bandung, Indonesia
erlangga.setyawan@gmail.com, didadiah@gmail.com, kamil.antonabdulbasah@gmail.com

*Abstract* - **Train scheduling is one of the tactical planning of a railway service company. The challenge of railway scheduling is when the type of track that is passed is a partial track double track. If the scheduling is not optimal, will cause the idle time, where the train must wait for the intersection. This research purpose mathematical model for train scheduling problem in partial double track rail line to minimize idle time. We consider three criteria in developing this mathematical model: (1) Coupling / uncoupling wagon to minimize deadhead trip; (2) Fixed block signaling; (3) Rail and station capacity; (4) Meeting and passing in partial double track. To test our model, we used actual data in Indonesian Railway, especially in shout track railway, track from Bandung - Yogyakarta City. Using this mathematical model, our purposed model can minimize total idle time.**

*Keywords* – **Railway Optimization, Railway Scheduling, Operation Research.**

## I. INTRODUCTION

The high number of regular train departures required optimal rail scheduling to minimize travel time. Basically, the total travel time of rail *i* in segment *j* is the difference from leaving time of train *i* in segment *j* ($e_{ij}$) with leaving time of train *i* in segment *j* ($s_{ij}$). Total travel time consists of three components, i.e., the standard travel time, travel time tolerance and idle time. Standard travel time ($t_{ij}^0$) is the standard travel time required by train *i* to complete the journey on a segment *j*. The travel time of tolerance ($T_{ij}$) is the additional time or tolerance time of train in passing through a segment. The travel time of tolerance is required because the train runs on a segment at an unstable speed. If the travel time of rail *i* in segment *j* ($e_{ij} - s_{ij}) \geq t_{ij}^0 + T_{ij}$, the time difference between $e_{ij} - s_{ij}$ with $t_{ij}^0 + T_{ij}$ is idle time ($\rho_{ij}$). The idle time of the train is divided into two, that is, the idle time when the train is waiting at the intersection and idle time when the train will enter segment *j*.

Scheduling performance of regular train which has been owned by Indonesia Railway not yet optimal, especially on the south track. This is evidenced by the train schedule still has a high total idle time on each railway class. Based on the priority level, regular trains are divided into five classes that indicate the priority of trains, i.e., first class, second class, third class, fourth and fifth class. The smaller the class, the higher the priority of the train. From the results of the problem studies, schedule performance and idle time data on the south track for each priority class is shown in Figure I. 1.

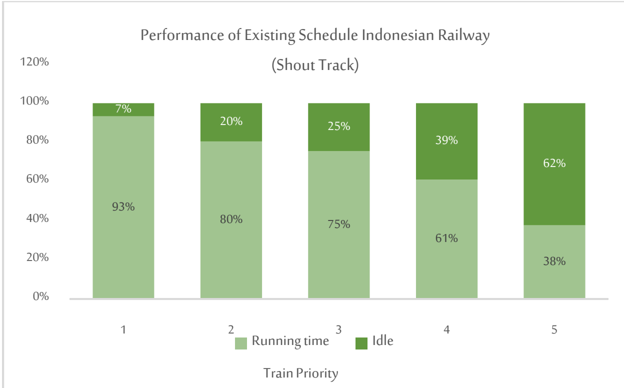


Fig. 1. Existing Schedule Indonesian Railway

To solve the high idle time on the Bandung - Yogyakarta railway track evaluated in Figure I. 1, the authors propose a model of rail scheduling by adopting multi-item multi-item scheduling on a partial dual track. In this study the authors will develop scheduling for railway cases in Indonesia, especially for the Bandung - Yogyakarta route to reduce the delay of train scheduling. The Bandung - Yogyakarta route was chosen to be the case study in this study because there are still 42% of single tracks and when viewed from topographical point of view, it is the track that has the most differences in the speed of the train in each segment. This stage will result in a more optimal new train schedule by minimizing total idle time. The scheduling model to be developed by the researcher is multi-item scheduling on multi-criteria capacitated no-wait parallel machine job shop scheduling by adding a drum buffer rope border function. This limiting function is the development of the theory of constrain used to optimize the balance in production machinery [1].

## II. RELATED WORKS

The development of the proposed railway scheduling model in this study, refers to case studies on actual train scheduling of Indonesian Railway Company. Some of the basic models used as a reference for model development in this study are the train scheduling model developed by [2] and [3]. Previous study [2] developed a railway scheduling model on a partial dual track that aims to minimize rail travel time. The research is more focused on the mathematical model to regulate the movement of trains from double tracks to single tracks in multi-direction. With the scheduling model, the train will be spared from collisions when the displacement process is on a partial double track.

The scheduling model developed by [3] is a scheduling model on a partial dual track with regard to penalties when a train is scheduled to arrive at a station beyond the expectation of a consumer. With the model, the train will be scheduled in accordance with the travel time desired by consumers, where the shorter the time of the railway, the higher the value of customer satisfaction. This type of partial double track becomes the limiting on this model because the train will require a longer travel time when it passes a single track.

The railway scheduling model [4] was adopted to apply the process when the train made a change in the number of railway compositions. While the scheduling model [5] is used to arrange rail scheduling with respect to the ratio of the number of single tracks and the number of double tracks. This scheduling model will be refined in this thesis research by adding a new limiting variable, that is, the drum buffer rope used to reduce the density when the train passes a single track from a dual track. The scheduling model with the buffer rope drum barrier will be adopted from the model developed by [1]. Drum buffer rope is one of the theory of constraints that serves to prevent bottle neck from occurring in a process that has a lower capacity than other processes.

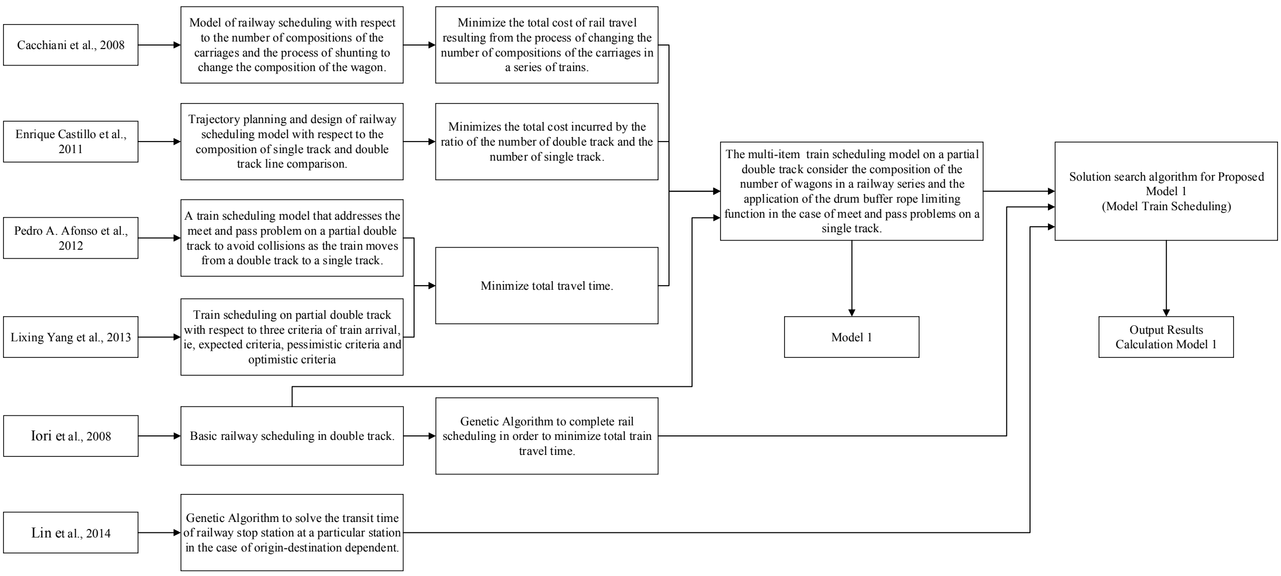


Fig. 2. Model Development of Railway Scheduling

Drum buffer rope will identify the number of reserve entities (trains) that are ready to be scheduled in a segment that has a low capacity. If the number of reserve trains has been reduced from the specified limit, the drum buffer rope will instruct the next train to depart from the initial station. This will avoid the buildup of segments with low capacity.

This study refers to several models of scheduling:

1. Research [6] uses the basic scheduling job shop scheduling to determine the optimal rail sequence in rail scheduling. The research was then developed by adopting a no-wait job shop scheduling algorithm where the algorithm is very focused on minimizing the delay in a production schedule.
2. The research was then developed by adopting a no-wait job shop scheduling algorithm where the algorithm is very focused on minimizing the delay in a production schedule [7] and [8].
3. Research [4] uses a parallel machine job shop scheduling approach with respect to variable amount of composition of passenger trains. The purpose of the research is to minimize the number of passenger train cars by adding and subtracting train cars according to the number of passengers at a certain stage.

Description of the development of the model to be used is as follows:

1. Multi-item job shop: used to optimize jobs simultaneously. Job is implied as a train that is dispatched to be processed on a machine (rail).
2. No-wait job shop: used to prioritize first class trains because they should not be interfered with the delays caused by trains other than the first class. In the event of a delay, then the train other than the first class is only allowed to be late because the first-class train has the highest priority.
3. Dynamic capacitated job shop scheduling: the number of passenger train cars may vary when train is traveling. This leads to differences in rail travel time in passing through a segment block.
4. Blocking job shop scheduling: used to optimize if one of the rails is impassable due to

interference. Trains will be diverted to other available rails. This condition is called the rescheduling backup track [9].

## III. METHODOLOGY

To develop mathematical model, we used some limitations and assumptions, i.e.:

1. Do not consider the rescheduling of the railway due to the occurrence of large disturbances and requires a long recovery time.
2. This study only uses historical data, not using actual data from ATP (automatic train protection) or ATC (automatic train control).
3. Not considering the use of locomotive types.
4. Do not consider the carriage of goods on a series of trains.

The objective function of regular rail scheduling is to minimize the total idle time of trains through Bandung - Yogyakarta. The idle time minimization evaluation process will be performed on each segment *j*, starting from the first segment *j=1*, where the train is dispatched from the starting station, until the train arrives at the end station *m*.

$$Minimize\ Z = \sum_{i=1}^{n}\sum_{j=1}^{m} \rho_{ij} \quad (1)$$

Constrains:

1. Leaving time train *i* in segment *j*

$$e_{(i,j)} \geq s_{(i,j)} + t_{ij}^{0} + \delta_{(i,j)} + 0{,}4\left(Wn_{ij}\frac{sd_{ij}}{1000}\right) + c_{(i,j)}\, max(0,\ C_{(i,j)}) \quad (2)$$

The train *i* will leave the segment *j* after the train *i* arrives in segment *j* $(s_{ij})$, plus the actual travel time $(t_{ij}^{0})$, the loading and or unloading time of the passenger, the time of the change of the composition of the railway wagon $(c_{i_{a_{ik}}})$. While $C_{ij}$ is the decision variable of a train to change the composition of the number of carriages (shunting process), $C_{ij}$ worth 0 if the replacement process of the composition of the carriage, $C_{ij}$ worth 1 if no replacement process of the composition of the carriage.

$$\delta_{i_{a_{ik}}} \begin{cases} \delta_{ij}^{min}, & if\ there\ is\ passanger's\ \ loading/unloading \\ \delta_{ij}^{max}, & otherwise \end{cases}$$

$$C_{ij} = \begin{cases} 1, & if\ Q_{kij} + \sum_{ijk} Q_{kij-1} > P_{kij}\ and\ w_{kij} < w_{ki}^{max} \\ 0, & otherwise \end{cases}$$

2. Station Capacity

$$\sum_{ijt} \gamma_{ijt} \leq b_j^{max} \quad (3)$$

Variable $b_j^{max}$ is used to define the maximum capacity of a station *j*. The decision variable $\gamma_{ijt}$ serves to indicate whether train *i* stops at station *j* at time *t* and is a binary variable. The variable $\gamma_{ijt}$ is 1 if the train *i* stops at station *j* at time *t* and 0 otherwise. The total number of trains *i* that stops at station *j* in time *t* $\sum_{ijt}\gamma_{ijt}$ must be less than or equal to the total capacity of a station.

3. Train Dwell Time While Entering Station (next segment)

$$s_{i_{a_{ik}}} \geq e_{i_{a_{ik-1}}} + d_{ij}^{min}(\alpha_{ijt}) \quad (4)$$

The arrival time of rail *i* in segment *j* is allowed to be greater than or equal to the time of departure of train *i* in *j-1* segment plus minimum dwell time tolerance $d_{ij}^{min}$, while $\alpha_{ijt}$ is binary variable if there is dwelling time when entering the station.

$$\alpha_{ijt} = \begin{cases} 1, & if \sum_{ijt} \gamma_{ijt} \geq b_j^{max} \\ 0, & otherwise \end{cases}$$

4. Minimum Headway (Opposite Direction)

$$s_{i_2 j} \geq e_{i_1 j} + h_j^0 x_{i_1,i_2,j} - M\,(1 - x_{i_1,i_2,j})$$
$$s_{i_1 j} \geq e_{i_2 j} + h_j^0 y_{i_1,i_2,j} - M\,(1 - y_{i_1,i_2,j}) \quad (5)$$

$$x_{i_1,i_2,j} = \begin{cases} 1, & if (PT_{i_1} < PT_{i_2}) \\ 0, & Otherwise \end{cases}$$

$$y_{i_1,i_2,j} = \begin{cases} 1, & if (PT_{i_1} > PT_{i_2}) \\ 0, & Otherwise \end{cases}$$

Since the railway station may be used in multi-direction, it will be the railroad scenario in the opposite direction, i.e., between the train going into the station downstream and the train going out to the downstream station.

5. Minimum Headway (Opposite Direction)

$$s_{i_2 j} \geq s_{i_1 j} + h_{i_1,i_2,} x_{i_1,i_2,j} - M\,(1 - x_{i_1,i_2,j})$$
$$s_{i_1 j} \geq s_{i_2 j} + h_{i_1,i_2,} y_{i_1,i_2,j} - M\,(1 - y_{i_1,i_2,j})$$
$$e_{i_2 j} \geq e_{i_1 j} + h_{i_1,i_2,} x_{i_1,i_2,j} - M\,(1 - x_{i_1,i_2,j}) \quad (6)$$
$$e_{i_1 j} \geq e_{i_2 j} + h_{i_1,i_2,} y_{i_1,i_2,j} - M\,(1 - y_{i_1,i_2,j})$$

Constraint (6) is an integration limiting function that limits the train going in or out to or from segment *j*. This constrain used to restrict and regulate the train leaving or entering the station in the same direction (same direction). The train boundary function that goes unidirectionally is simpler than the opposite train.

6. Meeting and Passing (Train Crossing)

$$s_{i_2(j+1)} \geq e_{i_1(j+1)} + M\,(1 - CR_{i_1,i_2,j})$$
$$s_{i_1 j} \geq e_{i_2 j} + M\,(1 - CR_{i_1,i_2,j}) \quad (7)$$

$$CR_{i_1,i_2,j} = \begin{cases} 1, & if (PT_{i_1} < PT_{i_2}) \\ 0, & Otherwise \end{cases}$$

This limiting function serves to set the train at the junction between single track and double track. Single track tracks can be traversed by rail in two directions, i.e., downstream direction (direction to Yogyakarta Station) and upstream direction (direction to Bandung Station).

The model proposed to solve the problems studied in this thesis research is a Non-Polynomial (NP) Hard Problem. This is because the proposed model has a high level of complexity as evidenced by the number of variables considered. This condition results in finding a solution using the exact optimization method, that is, mixed integer non-linear programming, unable to solve this problem because the computational time of its completion cannot be limited by polynomial time. Solution search is done using a metaheuristic approach algorithm. The metaheuristic algorithm limits the optimization process with decision problems, such as, the maximum number of iterations. The weakness of the metaheuristic algorithm is that the optimal solution produced is not as good as the exact method, such as, mixed integer non-linear programming.

The metaheuristic algorithm chosen to solve this scheduling problem is the Genetic Algorithm. Genetic Algebra performs several evaluations in finding optimal solutions, namely, fitness evaluation of the initial population and genetic evaluation. Chromosomes that have good fitness value are crossed on genetic evaluation with other chromosomes that have a good fitness value, so as to produce superior children. The results of the crossing process are evaluated again. Child chromosomes that have poor fitness values are eliminated / mutated so that they are not mated with other chromosomes in the next iteration. Iterations stop after reaching the maximum number of generations (number of iterations) that have been defined. This condition causes the Genetic Algorithm to produce an optimal solution that is better when compared to other metaheuristic algorithms.

## IV. RESULTS

### A. Transit Time Analysis

Improvements to rail scheduling are done to minimize the idle time used by trains to cross and queue when trains will enter the station. The measurement of decreasing idle time is by comparing the average length of rail transit time at a station in each class between initial conditions and proposed conditions. Since it is a parameter, the passenger loading time between the initial conditions and the proposed conditions is a fixed value. The idle time is a variable whose value will change as it is affected by the scheduling improvements made. The smaller the idle time value generated, the better the scheduling performance has been proposed. Table 1 and table 2 shows the differences between existing and purposing average transit time.

TABLE 1. Existing Average Transit Time

| Train Priority | Number of Train | Average Transit Time |
|---|---|---|
| 1 | 6 | 4,166666667 |
| 2 | 13 | 5,793040293 |
| 3 | 20 | 6,212200855 |
| 4 | 65 | 7,542305942 |
| 5 | 48 | 7,986805556 |

TABLE 2. Purposing Average Transit Time

| Train Priority | Number of Train | Average Transit Time |
|---|---|---|
| 1 | 6 | 3,775 |
| 2 | 13 | 5,034798535 |
| 3 | 20 | 5,404337607 |
| 4 | 65 | 7,006472203 |
| 5 | 48 | 7,817361111 |

### B. Idle Time Analysis

In accordance with the proposed rail scheduling model, to achieve the minimum idle value is by changing the decision variables in the form of departure and train arrival times in each segment. The correct combination of train scheduling sequences will minimize railway waiting time for crossing when moving from double track to single track and minimizing railway waiting time when it enters the station. To cross the rail, a train with a lower priority class, will wait for a higher priority rail to pass on a single track. Therefore, trains that have high priority, have an average value of idle time is smaller than the lower priority. Table 3 and Table 4 show the existing idle time values and the idle time purposing values of the optimized scheduling using the proposed model.

TABLE 3. Existing Idle Time

| Train Priority | Idle Percentage | |
|---|---|---|
| | Idle Minimum | Idle Maximum |
| 1 | 4% | 9% |
| 2 | 8% | 27% |
| 3 | 12% | 41% |
| 4 | 15% | 72% |
| 5 | 18% | 72% |

TABLE 4. Purposing Idle Time

| Train Priority | Idle Percentage | |
|---|---|---|
| | Idle Minimum | Idle Maximum |
| 1 | 1% | 6% |
| 2 | 1% | 21% |
| 3 | 4% | 26% |
| 4 | 3% | 54% |
| 5 | 17% | 59% |

### C. Makespan Analysis

The calculation of the makespan for the proposed condition is equal to the calculation of the makespan value of the initial condition. Scheduled train departure schedules, ordered starting from the earliest to the very latest departure schedule. The next step is to calculate the makepan value on the proposal schedule.

The total makepan value on the scheduling of the proposed conditions is 27 hours 4 minutes. When compared with the initial scheduling pattern, there is a decrease in the makespan time by 13.94%.

## V. CONCLUSION

The proposed model is a train scheduling model. The model can improve the initial scheduling performance of the Indonesian Railway Company. The improvement of the proposed scheduling model uses the no-wait method of job-shop scheduling on a parallel machine by adopting the theory of constrain in the form of a drum buffer rope. The results of the scheduling improvements that have been done can improve the initial scheduling of Indonesian Railway Company by 46.32%.

For further research, while developing rail scheduling, consider the following variables:

1. Highway density. The higher the rail utility used, the higher the congestion on the highway.
2. Taking into account the amount of rolling stock inventory, both locomotives and passenger cars.

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